FAR BEYOND

MAT122

Optimization

Stony Brook University

Find Extrema w Differentiation – Application

ex. A hobby store has 20 ft of fencing to fence off a rectangular area in the corner of a room to display an electric train setup.

What are the dimensions that will maximize this area?

What is the maximum area?

$$x + y = 20 \quad \Rightarrow \quad y = 20 - x$$

$$A = xy \quad \Rightarrow \quad A = x(20 - x)$$

Want: $A' = 0$

$$= 20x - x^2$$

$$A' = 20 - 2x = 0$$ solve for x

$$20 = 2x$$

$$\boxed{x = 10 \text{ feet}} \Rightarrow y = 20 - 10$$

$$\boxed{y = 10 \text{ feet}}$$

$$\Rightarrow A = 10 \cdot 10$$

$$\boxed{A = 100 \text{ sq ft}}$$

Follow up Question:

Is x a minimum or a maximum?

$A'' = -2$ $\quad$ A is always concave down

A has a maximum

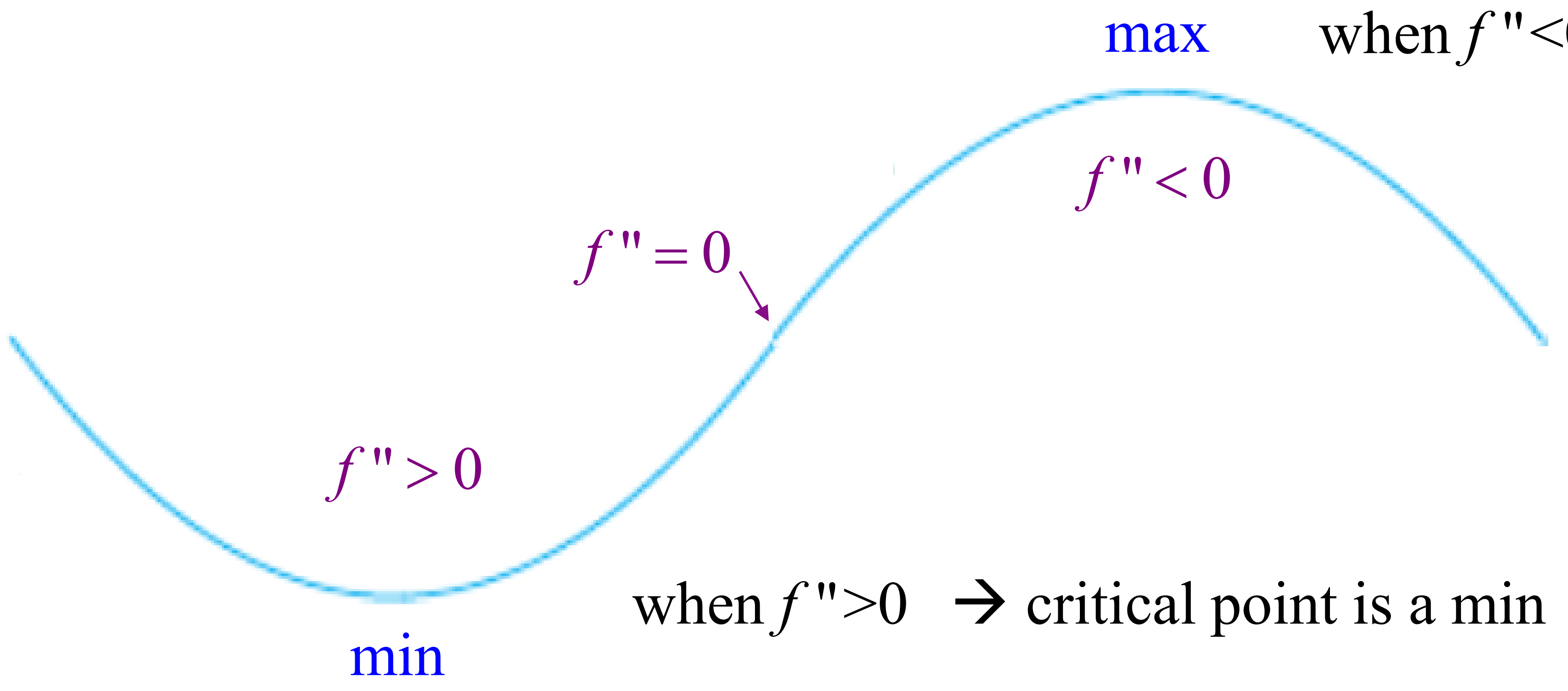
Meanings of Derivatives – Review #2

Concavity

If $f''> 0$ on an interval then f is **concave up** on that interval.

If $f''< 0$ on an interval then f is **concave down** on that interval.

when $f''=0$ f can have an inflection point

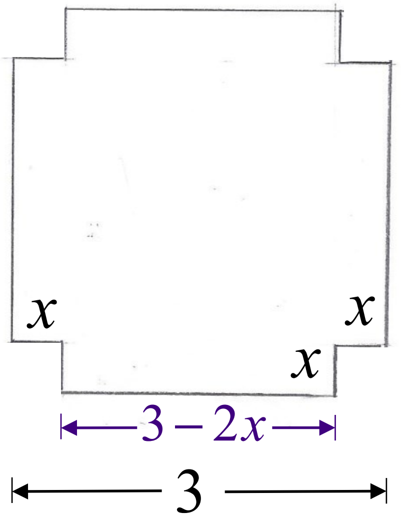


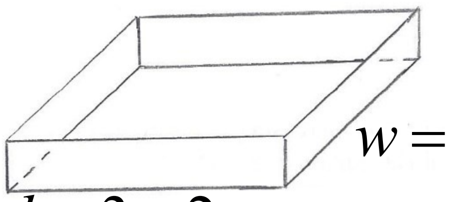
when $f''<0$ → critical point is a max

when $f''>0$ → critical point is a min

Optimize the Size of Square Packaging

Find the largest volume of a box constructed from a square piece of cardboard, 3 feet wide, by cutting out a square from each of the corners and bending up the sides. The box has an open top.





$$V_{box} = lwh \qquad square \Rightarrow l = w$$

$$V(x) = (3-2x)(3-2x)\,x \quad \text{FOIL}$$

$$= (9-12x+4x^2)\,x$$

$$= 9x - 12x^2 + 4x^3$$

$$V'(x) = 9 - 24x + 12x^2 = 0 \quad \text{reorder terms}$$

$$12x^2 - 24x + 9 = 0$$

solve for x $\quad 3(4x^2 - 8x + 3) = 0$

$$3(2x-3)(2x-1) = 0$$

$$x = \frac{3}{2}\,,\quad x = \frac{1}{2}$$ which yields a maximum?

3

$2x-3$

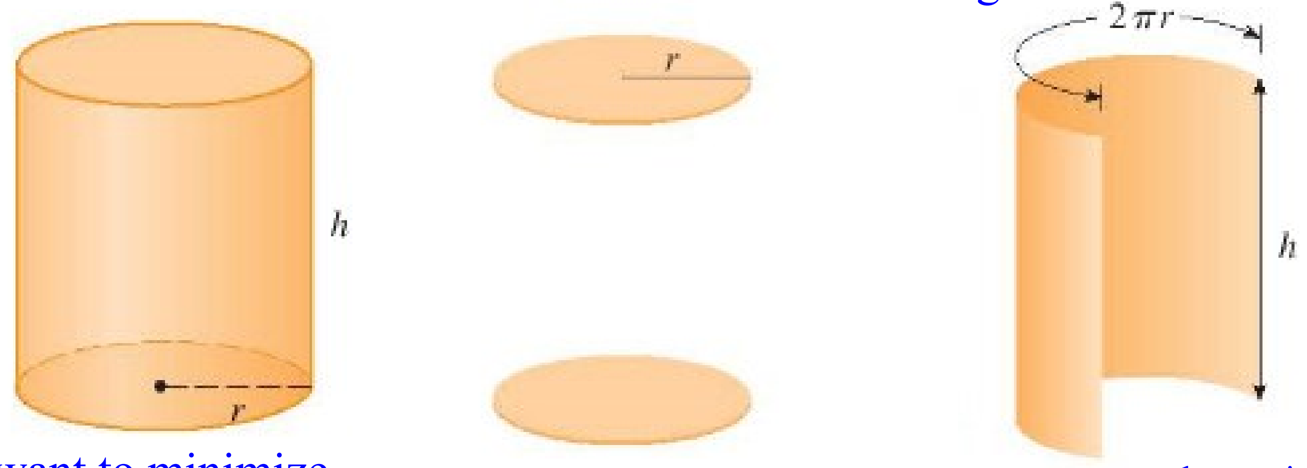
$2x-1$

Largest volume, $2\,ft^3$, occurs when $x = 1/2$

Optimize the Size of Cylindrical Packaging

A cylindrical can is made to hold 1000 cm^3 of oil. Find the dimensions that will minimize manufacturing costs.

base of rectangle = circle's circumference



want to minimize SURFACE AREA

$$SA_{top/bottom} = 2\pi r^2$$

base * height

$$SA_{side} = (2\pi r)h$$

write h in terms of r:

use volume formula

$$V_{cylinder} = \pi r^2 h = 1000$$

$$\therefore h = \frac{1000}{\pi r^2}$$

$$SA_{cylinder} = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$= 2\pi r^2 + 2000r^{-1}$$

Optimize the Size of Cylindrical Packaging (cont'd)

A cylindrical can is made to hold 1000 cm³ of oil. Find the dimensions that will minimize manufacturing costs.

want to minimize
SURFACE AREA

$$V_{cylinder} = \pi r^2 h = 1000$$

$$\therefore h = \frac{1000}{\pi r^2}$$

$$SA = 2\pi r^2 + 2000r^{-1}$$

$$(SA)' = 4\pi r - 2000r^{-2} = 0$$

$$4\pi r = \frac{2000}{r^2}$$

$$4\pi r^3 = 2000$$

$$r^3 = \frac{2000}{4\pi} = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}}$$

now solve for h:

$$h = \frac{1000}{\pi \sqrt[3]{500/\pi}^{\,2}}$$

$1000 = 2 * 500$

$$= \frac{1000}{\frac{\pi \sqrt[3]{500}^{\,2}}{\sqrt[3]{\pi}^{\,2}}} = \frac{2 \cdot 500^1}{\sqrt[3]{\pi} \cdot 500^{2/3}} = 2 \cdot \frac{\sqrt[3]{500}}{\sqrt[3]{\pi}}$$

$$\frac{\pi^1}{\pi^{2/3}} = \pi^{1/3} = \sqrt[3]{\pi}$$

$$\frac{500^1}{500^{2/3}} = 500^{1/3}$$

$$= 2 \cdot \sqrt[3]{\frac{500}{\pi}}$$

$$h = 2r$$

Can's dimensions are optimal when radius is $\sqrt[3]{500/\pi}\ cm$ and height is twice the radius.